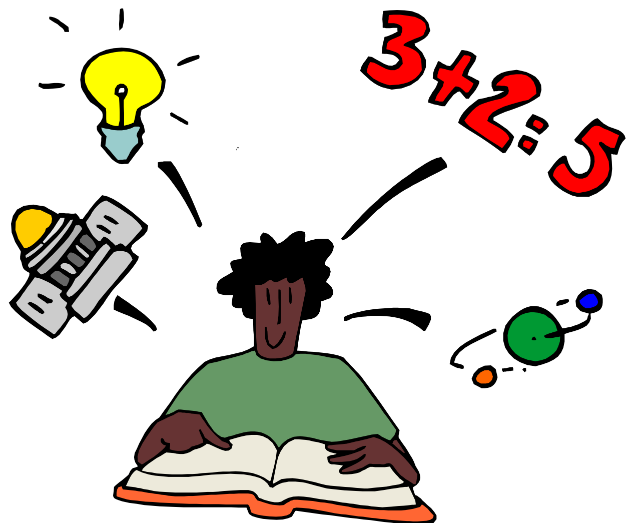
Millfields C E Primary School

Faith, Respect, Courtesy & Endeavour



Calculation Policy

June 2020

<u>Millfields C E Primary School</u>

<u>Written and Mental Calculations Policy</u>

Article 28: Every child has the right to an education, Article 29: Education must develop every child's personality, talents and abilities to the full.

This policy was developed using ideas from the 'Progression Through Calculation Guidance' by the Maths Hubs and aims to ensure consistency in the mathematical written methods and approaches to calculation across years 1-6. F1 and F2 needs will be met through the Statutory framework for EYFS, Development Matters and Early Learning Goals. Teachers will use the White Rose Reception Guidance to support teaching for mastery, focussing on the 5 principles of counting;

- The one-one principle
- The stable order principle
- The cardinal principle
- The abstraction principle
- The order-irrelevance principle

Wherever possible, it is important for teachers to create real life contexts for learning in maths. As part of a child's learning in calculation, they need to be taught how to select the best method according to the numbers. This policy is separated into each of the 4 areas of calculation and exemplifies good practise for combining the use of concrete, pictorial and abstract (CPA) strategies so that children become confident and secure with their understanding of both written and mental methods. This policy should be used to ensure that CPA is being effectively embedded across school.

CPA is an effective approach that helps children to develop deep conceptual understanding and secure solid foundations for future learning. Psychologist Jerome Bruner, suggests that concrete, pictorial and abstract are the three steps (or representations) necessary for pupils to develop understanding of a concept. He states that reinforcement is achieved by going back and forth between these representations.

Concrete - In this stage the children are introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component and is the foundation for conceptual understanding.

Pictorial – When children have sufficiently understood the 'hands-on' experiences performed, they can relate them to representations, such as a diagram or picture of the problem.

Abstract- In this symbolic stage, children are now capable of representing problems by using mathematical notation, for example: 10 ÷ 2 = 5. Children only use abstract numbers and figures when they have enough context to understand what they mean.

Teachers aim to provide children with efficient counting strategies, and a secure knowledge of number facts and place value, and use these to develop the four operations. The children will be encouraged to look at a calculation/problem and then decide which is the best method to use, for example: pictures, mental calculation (with or without jottings) or a structured recording.

Our Intent

Our curriculum has been coherently planned and sequenced to ensure that each and every child can “live life in all its fullness” by offering ambitious, awe-inspiring learning experiences with Christian values at the heart of everything we do. We intend to provide broad and balanced learning experiences for our children that not only focus on specific knowledge, skills and understanding as set out in the National Curriculum but also provide opportunities for children to develop the skills needed to prepare them for challenges beyond the classroom and for later life. We want our children to feel empowered in all aspects of their learning through "Faith, Respect, Courtesy and Endeavour "and aim to achieve this by encouraging our children to become;

- Self-managers
- Effective participators
- Resourceful thinkers
- Reflective learners
- Independent enquirers
- Team workers

Our bespoke curriculum is constantly evolving in order to meet the unique needs and interests of all our children. We feel it is important for our children to develop both a sense of belonging within their own local community and develop a growing awareness of where they fit within the wider world by exploring a range of national and global issues.

We have identified a clear progression of skills within all aspects of the curriculum and are able to use this information to feel confident in understanding how our children learn, what our children already know and what they need next in order to be well prepared for the next stage of their learning journey. In addition to building secure relationships with our children, we value the importance of knowing the skills and expertise of our staff and use this information to work as a team in order to maximise the variety and quality of the learning experiences we provide.

We endeavour for all our children to leave Millfields C.E. Primary School equipped with the knowledge and skills needed to be prepared for whatever comes next in their life. In order to achieve this, we feel it is vital for children to have developed the strength of character and confidence needed to make decisions and a rich vocabulary in order to communicate and articulate their thinking. If our children are to succeed as lifelong learners, they will have no limits in their curiosity, thirst for knowledge and desire for new experiences.

Implementation

We implement our curriculum through a range of theme based approaches which help us to teach and offer our children imaginative learning projects in which they can use and apply their knowledge and skills. However, some subjects and specific knowledge and skills are taught discreetly to ensure that the children progress throughout all key stages and are well rounded citizens with a range of skills for use in future life. Teachers have access to an extensive range of CPD and we use our staff skills to support each other in developing a progressive age appropriate curriculum. We evaluate our curriculum through rigorous evidence from pupil voice, questionnaires and through developing 'their' interests. We take into consideration local, national and global events and incorporate these into our curriculum. The curriculum is designed and delivered to allow pupils to use and transfer key knowledge and skills and commit these to long term memory as well as promoting the importance of good verbal communication skills with a vast range of vocabulary to exceed in any challenge. At Millfields C.E. Primary School, the children receive many different challenges across the curriculum and are able to access a variety of enrichment activities. Educational visits and visitors are a key feature of the curriculum; the school subsidises a range of these to enrich the curriculum for all. Throughout all aspects of our teaching and learning, the teacher will consistently use a wide range of assessments as well as the children's prior learning to embed and deepen the children's learning.

Our long term aim is …

For our children to confidently approach a range of problems or calculations in a variety of contexts, selecting appropriate, efficient methods.

Addition and Subtraction

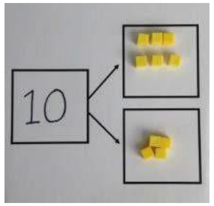
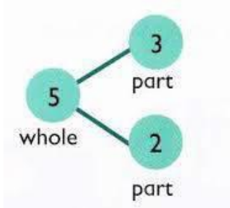
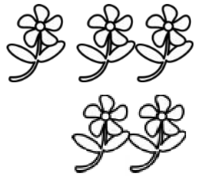
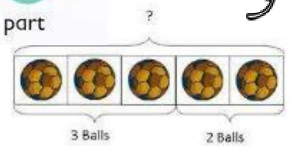
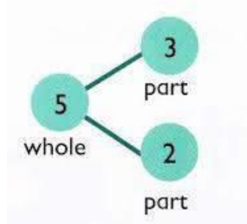
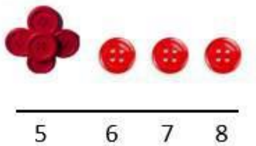
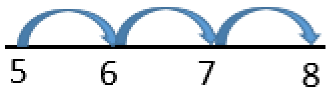
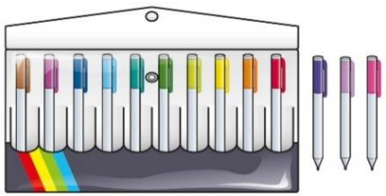
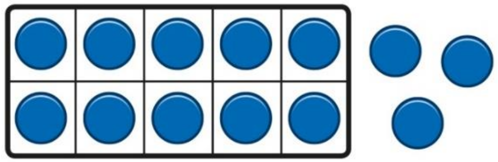
Key Stage 1	Lower Key Stage 2	Upper Key Stage 2
Children first learn to connect addition and subtraction with counting. They soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. Children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 − 3 and 15 − 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods. In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not think it is appropriate to include it until Year 3.	In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the progression of steps helps children to develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved. This will enable them to apply these skills accurately and efficiently to later calculations. Children should be encouraged to compare mental and written methods for specific calculations and should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.	Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

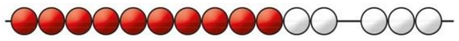
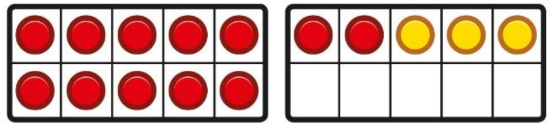
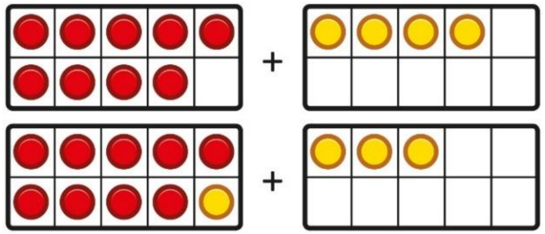
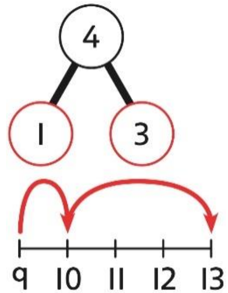
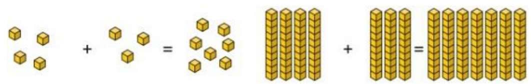
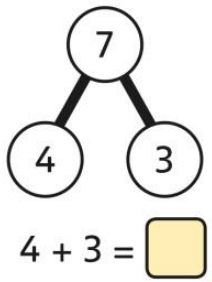
ADDITION

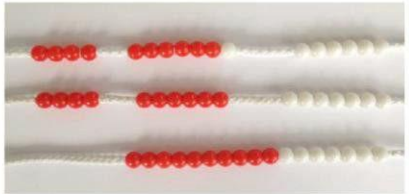
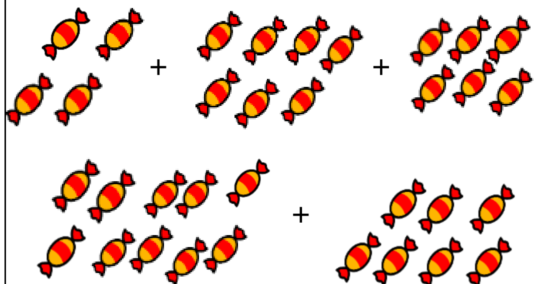
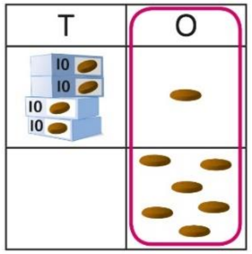
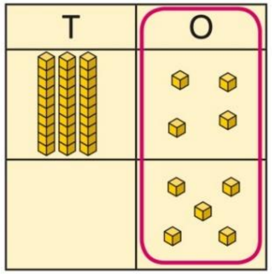
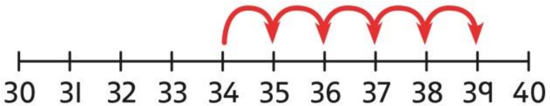
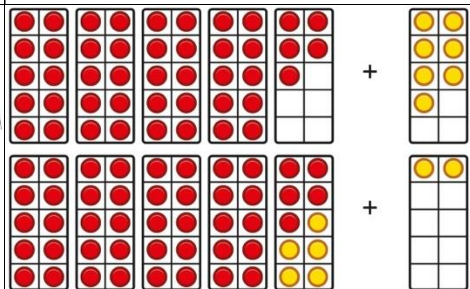
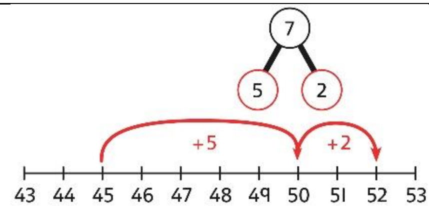
Vocabulary linked to Addition:
add, more, sum, total, make, greater, plus, addition, increase, whole, part, number bond, partition, place value, ones, ten, tens, hundreds, thousands, ten thousand, hundred thousand, million, column method, decimal

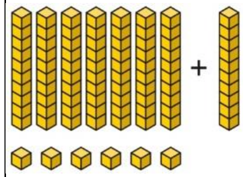
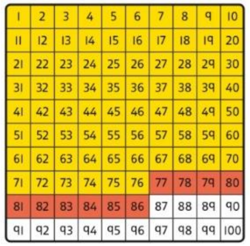
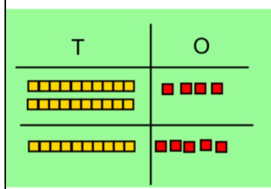
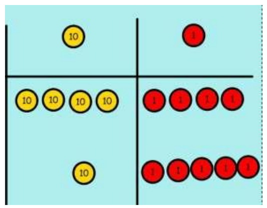
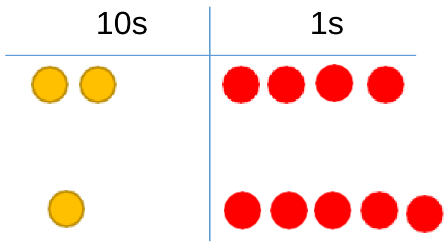
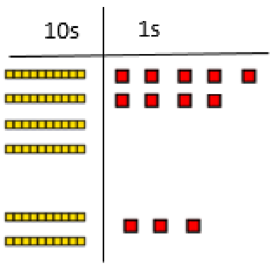
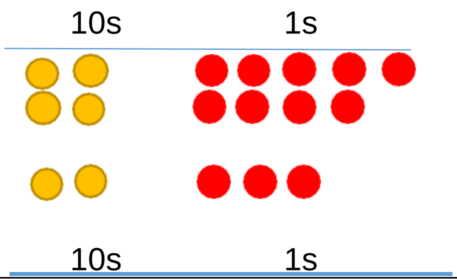
Common misconceptions:

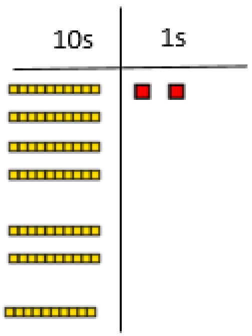
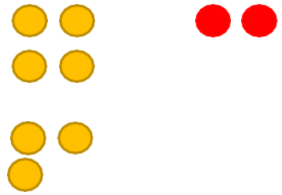
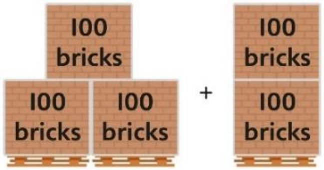
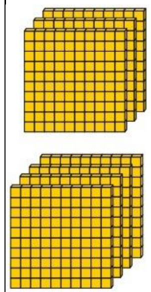
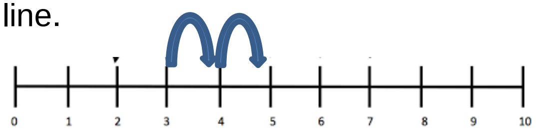
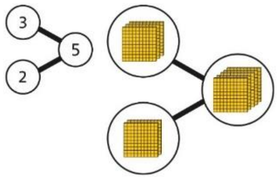
- Not estimating/making unrealistic estimations first to see if their answer ‘makes sense’
- Setting out when working in columns - confusion over the place value
- Confusion of ‘teen’ and ‘ty’
- Use of number line – count start number so calculation is out by 1

	Objective	Concrete	Pictorial	Abstract
Year 1 Addition	**Number bonds of 5, 6, 7, 8, 9 and 10**	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3  Use the part-part-whole diagram (as shown above) to move into the abstract.
	Counting on	 Start with the larger number on the bead string and then count on the smaller number 1 by 1 to find the answer. 	Use a number line to count on in ones. 	5 + 3 = 8
	Understanding teen numbers as a complete 10 and some more	Complete a group of 10 objects and count more.  13 is 10 and 3 more.	Use a ten frame to support understanding of a complete 10 for teen numbers.  13 is 10 and 3 more.	1 ten and 3 ones equal 13. 10 + 3 = 13

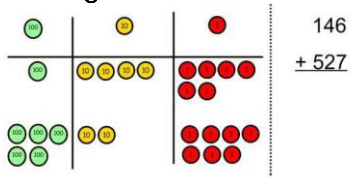
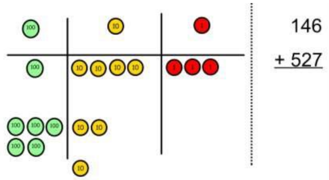
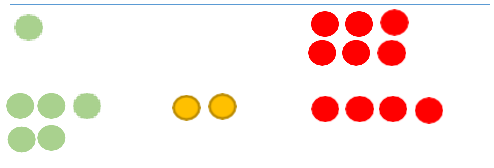
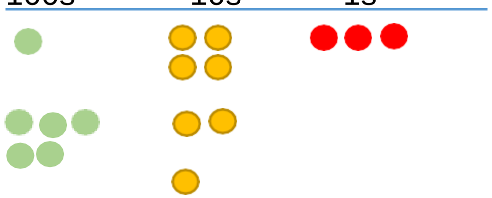
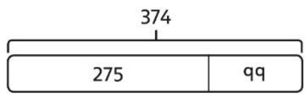
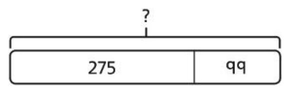
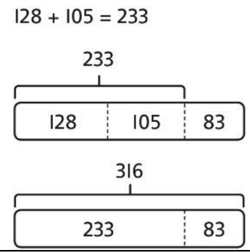
	Objective	Concrete	Pictorial	Abstract
Year 1 Addition	**Adding the 1s**	Use bead strings to recognise how to add the 1s to find the total efficiently.  *2 + 3 = 5* *12 + 3 = 15*	Represent calculations using ten frames to add a teen and 1s.  *2 + 3 = 5* *12 + 3 = 15*	Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. *3 + 5 = 8* *So, 13 + 5 = 18*
Year 1 Addition	**Bridging the 10 using number bonds**	Use a bead string to complete a 10 and understand how this relates to the addition.  *"7 add 3 makes 10.* *So, 7 add 5 is 10 and 2 more."*	Use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10. 	Use a part-whole model and a number line to support the calculation.  *9 + 4 = 13*
Year 2 Addition	**Adding 10s**	Use known bonds and unitising to add 10s.  I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.	Use known bonds and unitising to add 10s.  I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.	Use known bonds and unitising to add 10s.  4 + 3 = 7 4 tens + 3 tens = 7 tens 40 + 30 = 70

Objective	Concrete	Pictorial	Abstract
Adding 3 single digit numbers	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	4 + 7 + 6 = $\boxed{10}$ + $\boxed{7}$ 10 = $\boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.
Adding a 1-digit number to a 2-digit number not bridging a 10	Add the 1s to find the total. Use known bonds within 10.  41 is 4 tens and 1 one. 41 add 6 ones is 4 tens and 7 ones. 	 34 is 3 tens and 4 ones. 4 ones and 5 ones are 9 ones.  The total is 3 tens and 9 ones.	Understand the link between counting on and using known number facts. Use known number bonds to improve efficiency and accuracy.  *34 + 5 = 39* *or* T O 3 4 + 5 9
Adding a 1-digit number to a 2-digit number bridging 10	 There are 4 tens and 5 ones. *"I need to add 7. I will use 5 to complete a 10, then add 2 more."*		 *7 = 5 + 2* *45 + 5 + 2 = 52*

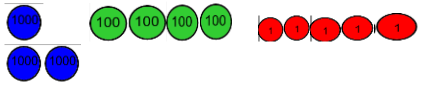
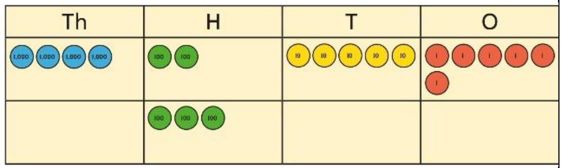
Year 2 Addition	Objective	Concrete	Pictorial	Abstract
	Adding a multiple of 10 to a 2-digit number	Use items that come in packs of 10 and single ones.  27 is 2 tens and 7 ones. 50 is 5 tens. There are 7 tens in total and 7 ones. So, 27 + 50 is 7 tens and 7 ones.	Images of dienes represent tens and ones.  66 is 6 tens and 6 ones. 66 + 10 = 76  A 100 square can support this	37 + 20 =? 30 + 20 = 50 50 + 7 = 57 37 + 20 = 57
	Column method (2 digit +2 digit) without regrouping	Add together the ones first then add the tens. Use the dienes blocks first before moving onto place value counters. 24 + 15 =  44 + 15 = 	After physically using the dienes blocks and place value counters, children can draw the counters to help them to solve additions.  	24 + 15 = 39 T O 2 4 + 1 5 3 9
	Column method with regrouping	Make both numbers on a place value grid.  	Using place value counters, children can draw the counters to help them to solve additions.  	40 + 9 20 + 3 60 + 12 = 72 Start by partitioning the numbers before formal column to show the exchange.

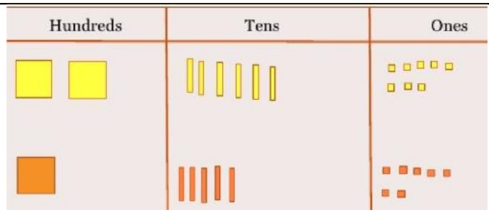
		 Add up the ones and exchange 10 ones for 1 ten.		
Year 3 Addition	**Adding 100s**	Use known facts and unitising to add multiples of 100.  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	 *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	Represent the addition on a number line.  Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*

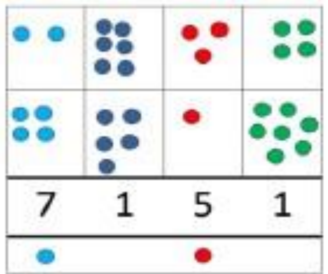
Year 3 Addition

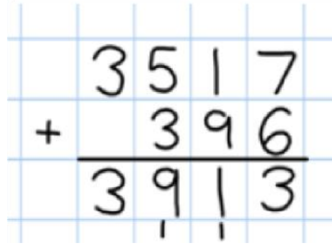
Objective	Concrete	Pictorial	Abstract
Column method with regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten.  As children move on to decimals, money and decimal place value counters can be used to support learning.	100s 10s 1s  100s 10s 1s  Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.	100 + 40 + 6 500 + 20 + 7 600 + 70 + 3 = 673 As the children progress, they will move from the expanded to the compacted method. 146 + 527 673 1 As the children move on, introduce both decimals with the same number of decimal places and different. Money can be used here.
	Note: Children should also study examples where exchange is required in more than one column, for example *185 + 318 =?*		
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 =?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *"I will add 100, then subtract 1 to find the solution".* *128 + 105 + 83 =?* *"I need to add three numbers".* 

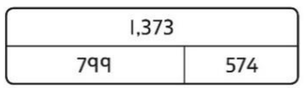
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="3">Year 4 Addition</td>
<td>Choosing mental methods where appropriate</td>
<td>Use unitising and known facts to support mental calculations, e.g.1,405 +2000 =?
Make 1,405 from pv equipment.

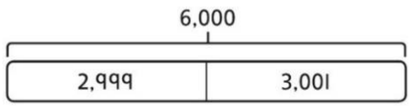
Add 2,000.

Then add the 1,000s.
1 thousand + 2 thousands = 3 thousands so 1,405 + 2,000 = 3,405</td>
<td>Use a place value chart as visual image.


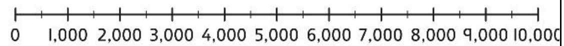
Add the 100s mentally:
200 + 300 = 500
So, 4,256 + 300 = 4,556</td>
<td>4,256 + 300 =?
2 + 3 = 5 so 200 + 300 = 500
4,256 + 300 = 4,556</td></tr>
<tr><td>Add numbers with up to 4 digits</td>
<td>

Continue to use dienes or place value counters to add; exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</td>
<td>

Draw representations using place value grid.</td>
<td>Continue from previous work to carry hundreds as well as tens.


Relate to money and measures.</td></tr>
<tr><td>Representing additions and checking strategies</td>
<td></td>
<td>Bar models can be used to justify mental methods and to represent addition during problem solving.


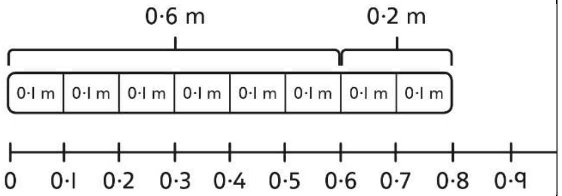
“I chose to work out 574 + 800, then subtract 1”.


“This is equivalent to 3,000 + 3,000.”</td>
<td>Use rounding and estimating on a number line to check the reasonableness of an addition.


912 + 6,149 =?
“I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.”</td></tr>
</table>

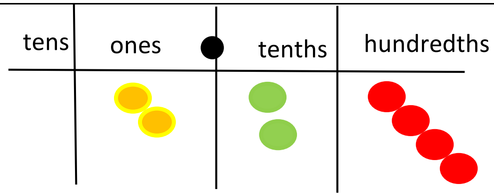
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="3">Year 5 Addition</td><td>Adding tenths</td><td>Link measure with addition of decimals.

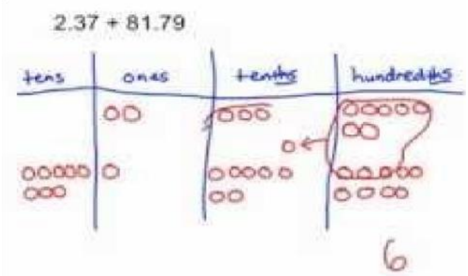
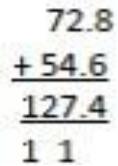
Two lengths of fencing are 0·6 m and 0·2 m.
How long are they when added together?

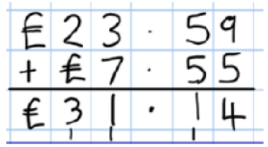
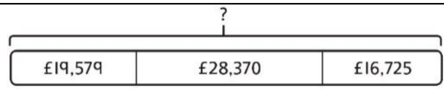
</td><td>Use a bar model with a number line to add tenths.


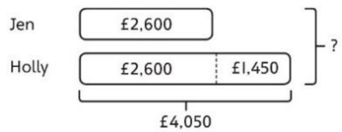
0·6 + 0·2 = 0·8
6 tenths + 2 tenths = 8 tenths</td><td>Understand the link with adding fractions.

$\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$

6 tenths + 2 tenths = 8 tenths
0·6 + 0·2 = 0·8</td></tr>
<tr><td>Add decimals with 2 decimal places, including money.</td><td>

Introduce decimal place value counters and model exchange for addition.</td><td>
</td><td>


</td></tr>
<tr><td>Representing additions</td><td></td><td>

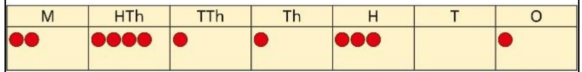
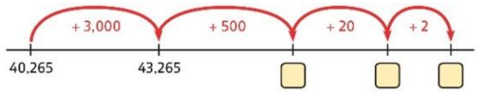
Bar models represent addition of two or more numbers in the context of problem solving.


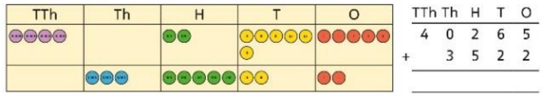

</td><td>Use approximation to check whether answers are reasonable.

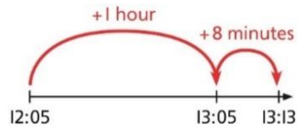
TTh Th H T O
2 3 4 0 5
+ 7 8 9 2
2 0 2 9 7

TTh Th H T O
2 3 4 0 5
+ 7 8 9 2
3 1 2 9 7

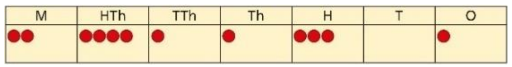
"I will use 23,000 + 8,000 to check."</td></tr>
</table>

<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 6 Addition</td><td>Comparing and selecting efficient methods</td><td>Represent 7-digit numbers on a place value grid and use this to support thinking and mental methods.

</td><td>Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.




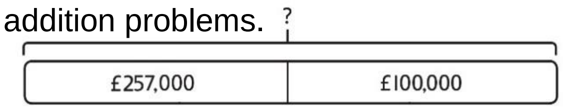
Use bar model and number line representations to model addition in problem-solving and measure contexts.

</td><td>Use column addition where mental methods are not efficient. Recognise common errors with column addition.
32,145 + 4,302 =?
TTh Th H T O
3 2 1 4 5
+ 4 3 0 2
3 6 4 4 7

TTh Th H T O
3 2 1 4 5
+ 4 3 0 2
7 5 1 6 5
“Which method has been completed accurately?”
“What mistake has been made?”
Column methods are also used for decimal additions where mental methods are not efficient.

H T O · Tth Hth
1 4 0 · 0 9
+ 4 9 · 8 9
1 8 9 · 9 8
1</td></tr>
<tr><td>Selecting mental methods for larger numbers where appropriate</td><td>Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.


2,411,301 + 500,000 =?

This would be 5 more counters in the HTh place.

So, the total is 2,911,301.
2,411,301 + 500,000 = 2,911,301</td><td>Use a bar model to support thinking in addition problems.


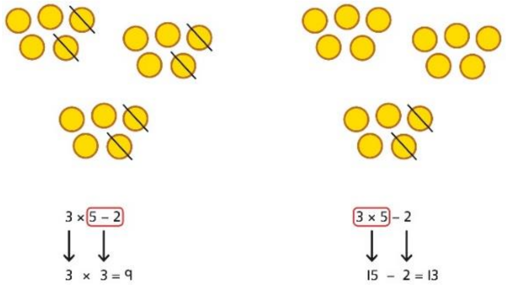
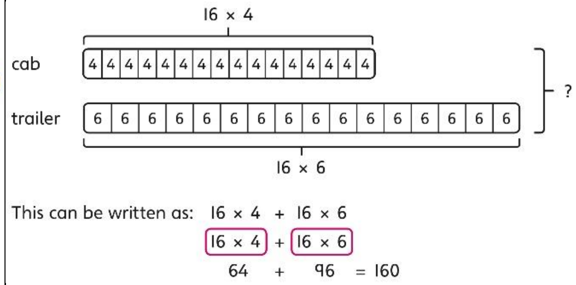
257,000 + 99,000 =?
“I added 100 thousands then subtracted 1 thousand.”
257 thousands + 100 thousands = 357 thousands
257,000 + 100,000 = 357,000
357,000 – 1,000 = 356,000
So, 257,000 + 99,000 = 356,000</td><td>Use place value and unitising to support mental calculations with larger numbers.

195,000 + 6,000 =?

195 + 5 + 1 = 201

195 thousands + 6 thousands = 201 thousands

So, 195,000 + 6,000 = 201,000</td></tr>
</table>

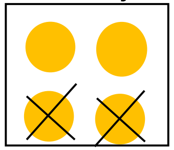
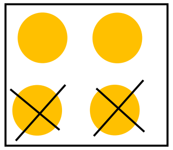
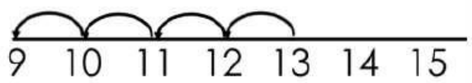
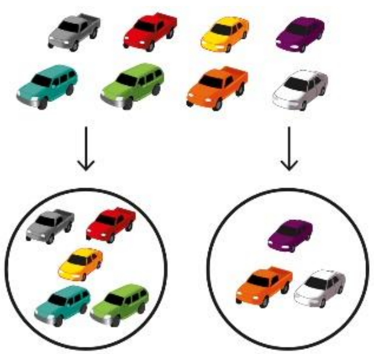
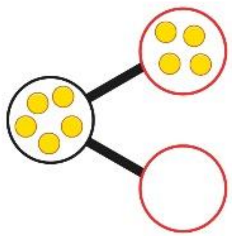
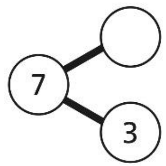
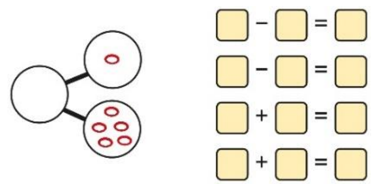
	Objective	Concrete	Pictorial	Abstract
	Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 − 2 =?* 	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. 	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*

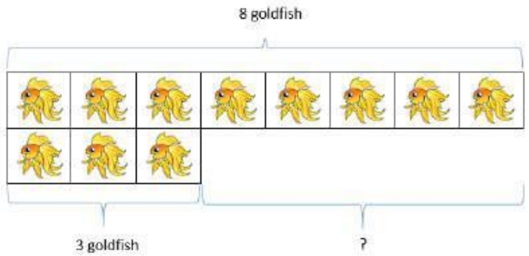
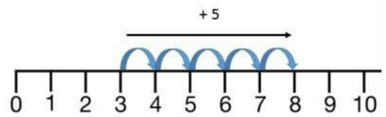
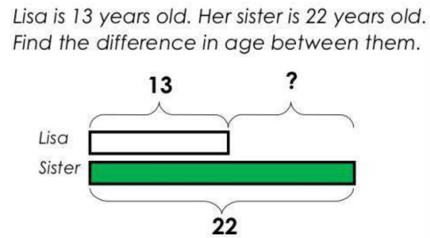
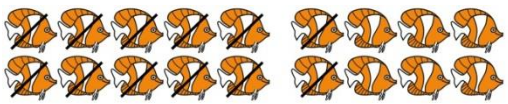
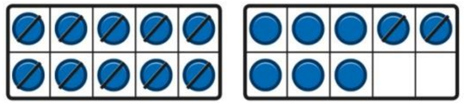
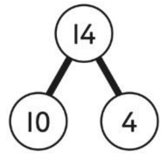
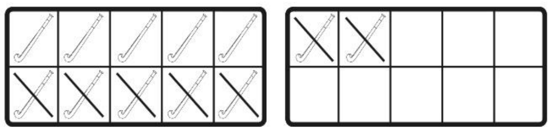
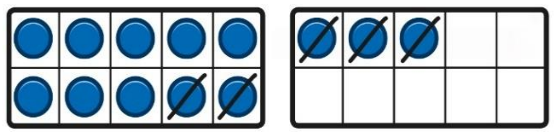
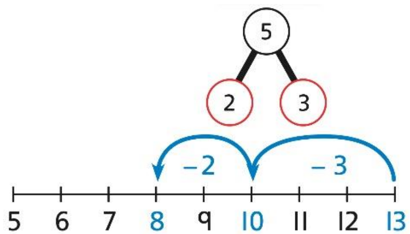
SUBTRACTION

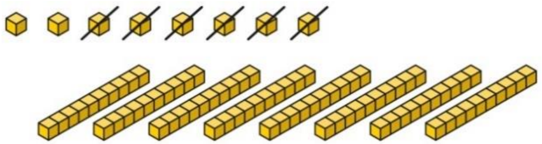
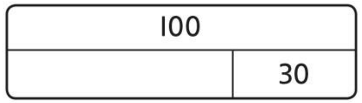
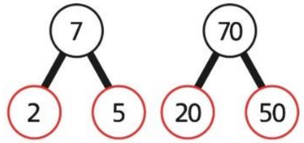
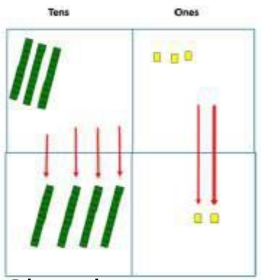
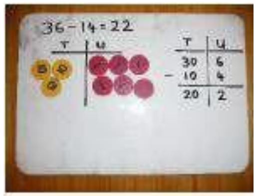
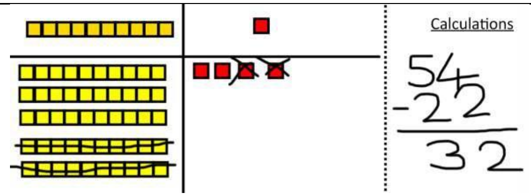
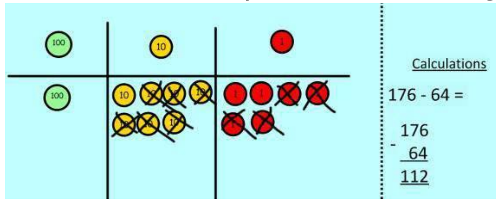
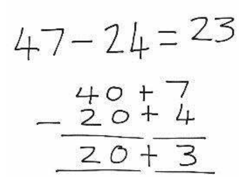
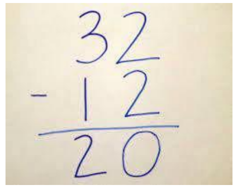
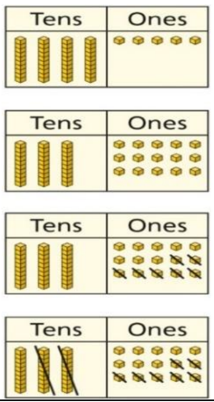
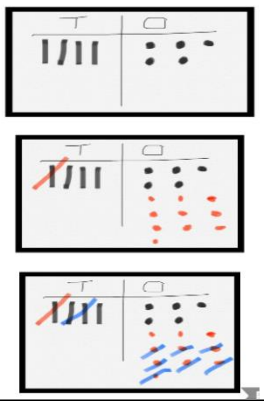
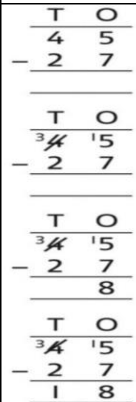
<u>Vocabulary linked to Subtraction</u>:
take, take-away, leave, left, fewer, less than, decrease, difference between, minus, subtract, subtraction, less, … more than, whole, part, exchange, partition, mental method, column method, ones, tens, hundreds, thousands

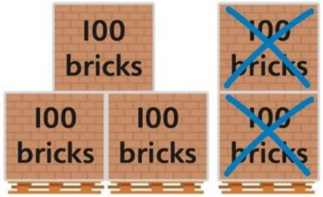
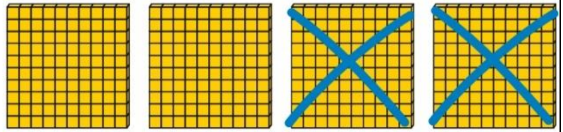
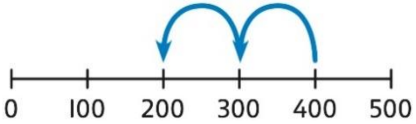
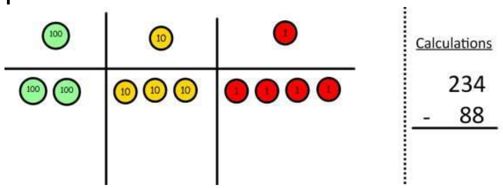
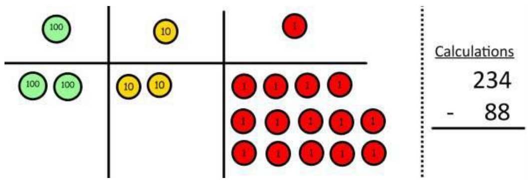
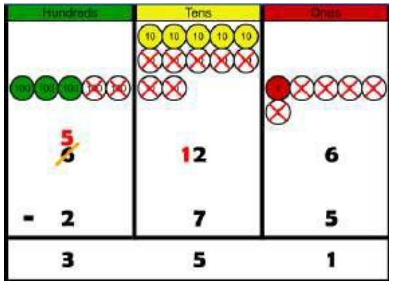
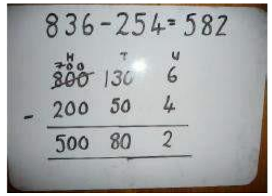
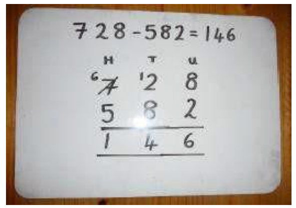
<u>Common misconceptions:</u>

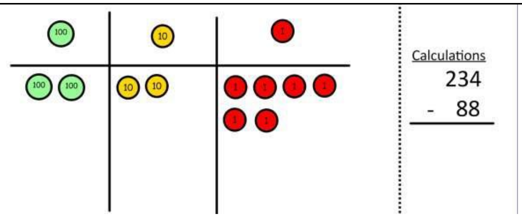
- Not estimating/making unrealistic estimations first to see if their answer ‘makes sense’
- Setting out when working in columns – confusion over the place value
- Confusion of ‘teen’ and ‘ty’
- Use of number line – count start number so calculation is out by 1
- Misunderstanding regarding place value and the concept of exchanging **T** for ones, **H** for **T**ens etc
- **Lack of understanding that when subtracting from a number that the answer will be smaller than the start number**
- **Children switch the digits around to be able to ‘do’ the calculation (believe it is commutative as with +/x)**

	Objective	Concrete	Pictorial	Abstract
Year 1 Subtraction	**Taking away ones**	Use physical objects, counters, cubes etc. to show how objects can be taken away. 4 – 2 = 2 	Cross out drawn objects to show what has been taken away. 4 – 2 = 2 	4 – 2 = 2
	Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 = 9	Count back on a number line or number track.  Start at the bigger number and count back the smaller number, showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
	Finding a missing part, given a whole and a part	Separate a whole into parts and understand how one part can be found by subtraction.  *8 − 5 =?*	Represent a whole and a part and understand how to find the missing part by subtraction.  $\boxed{5} - \boxed{4} = \boxed{}$	Use a part-whole model to support the subtraction to find a missing part.  *7 − 3 =?* Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model. 

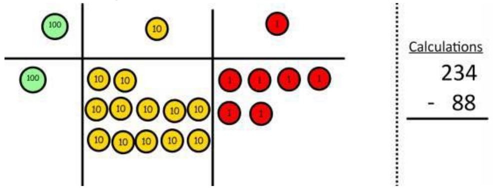
	Objective	Concrete	Pictorial	Abstract
Year 1 Subtraction	**Find the difference**	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference.	 Count on to find the difference.  Draw bars to find the difference between 2 numbers.	Hannah has 8 goldfish. Helen has 3 goldfish. Find the difference between the number of goldfish the girls have.
	Subtracting 10s and 1s	For example: 18 − 12 Subtract 12 by first subtracting the 10, then the remaining 2. 	18 − 12 Use ten frames to represent the efficient method of subtracting 12.  First subtract the 10, then subtract 2.	Use a part-whole model to support the calculation. *19 – 14=?* *19 − 10 = 9* *9 − 4 = 5* *So, 19 − 14 = 5* 
	Subtraction bridging 10 using number bonds	For example: 12 − 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.  *“7 is 2 and 5, so I take away the 2 and then the 5.”*	Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.*	Use a number line and a part-whole model to support the method. 13 − 5 

	Objective	Concrete	Pictorial	Abstract
Year 2 Subtraction	**Subtracting multiples of 10**	Use known number bonds and unitising to subtract multiples of 10.  8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.	Make/draw a bar model to show the relationship between bonds  *"10 − 3 = 7* *So, 10 tens subtract 3 tens is 7 tens."*	Use a part-whole model to relate known number bonds and unitising to subtract multiples of 10.  7 tens subtract 5 tens is 2 tens. 70 − 50 = 20
	Column method without regrouping	 75 – 42 = 33 Use dienes to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the dienes or place value counters alongside the written calculation to help to show working. 	 This will lead to a clear written column subtraction. 
	Subtracting a 2-digit number with exchange	 Use dienes to make the bigger number, then exchange 1 ten for 10 ones. Subtract the 1s. Then subtract the 10s	 Draw the dienes or place value counters. Exchange 1 ten for 10 ones. Subtract the 1s. Then subtract the 10s	 Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.

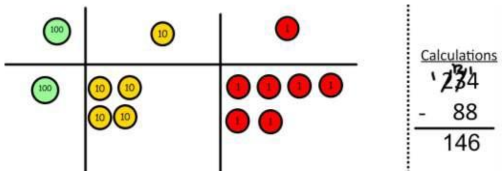
	Objective	Concrete	Pictorial	Abstract
Year 3 Subtraction	**Subtracting 100s**	Use known facts and unitising to subtract multiples of 100. Use objects/dienes/place value counters.  *“If 5 − 2 = 3 then 500 − 200 = 300”*	Draw the dienes to subtract multiples of 100.  *4 − 2 = 2* *400 − 200 = 200*	Understand the link with counting back in 100s.  *400 − 200 = 200* Use known facts and unitising as efficient and accurate methods. “I know that 7 − 4 = 3. Therefore, I know that 700 − 400 = 300.”
	See Year 2	**Column method without regrouping**		
	Column method with regrouping	Use dienes before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones, “can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.”  “Now I can subtract my ones.”	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out-clearly show the exchanges you make. When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.

<table>
<tr>
<td></td>
<td></td>
<td>


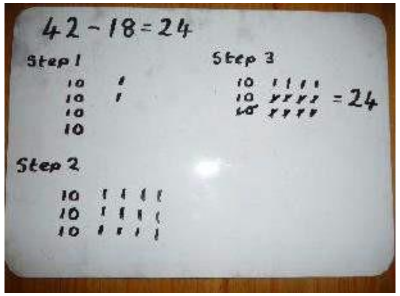
Now look at the tens, can I take away 8 tens easily? I need to exchange 1 hundred for 10 tens.



Now I can take away 8 tens and complete my subtraction.



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.
</td>
<td>
when to exchange/regroup.



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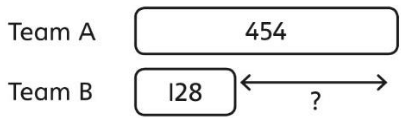
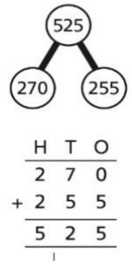
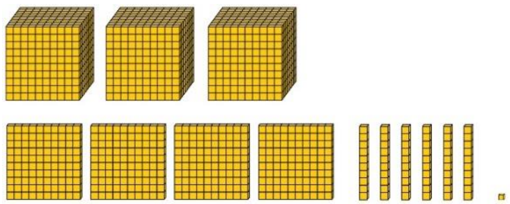
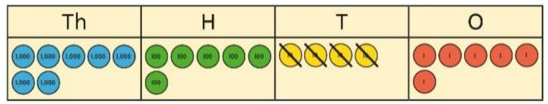
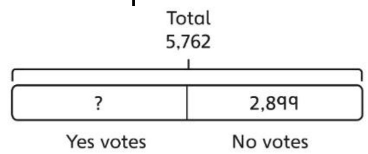
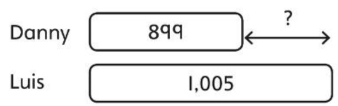
<td>

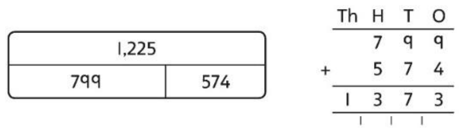
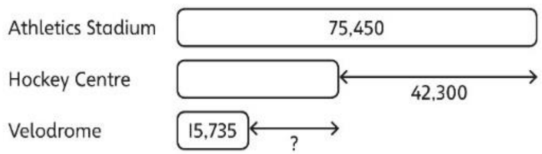
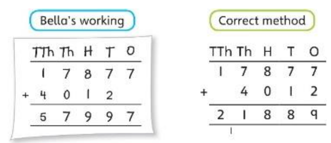
$$\begin{array}{rccccc} & & {}^{5} & {}^{12} & & {}^{1} \\ & 2 & \cancel{6} & \cancel{3} & . & 0 \\ - & & 2 & 6 & . & 5 \\ \hline & 2 & 3 & 6 & . & 5 \end{array}$$

</td>
</tr>
</table>

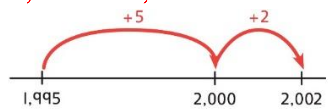
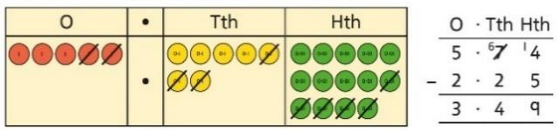
	Objective	Concrete	Pictorial	Abstract
Year 3 Subtraction	**Representing subtraction problems**		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Use alternative representations to check calculations and choose efficient methods. Use inverse operations to check additions and subtractions. The part-whole model supports understanding.  *"I have completed this subtraction.* *525 − 270 = 255* *I will check using addition."*
Year 4 Subtraction	**Choosing mental methods where appropriate**	Use place value equipment to justify mental methods.  *"What number will be left if we take away 300?"*	Use place value grids to support mental methods where appropriate.  *7,646 − 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 − 2,000* *3 thousands − 2 thousands = 1 thousand* *3,501 − 2,000 = 1,501*
	See Year 2 and Year 3	Year 4 should use the **column method** where mental methods are not efficient		

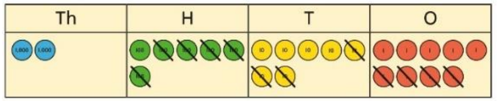
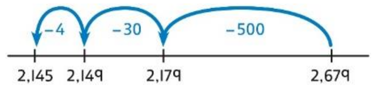
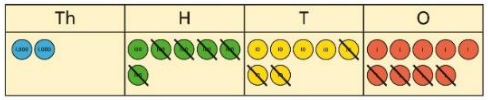
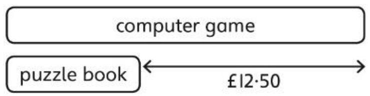
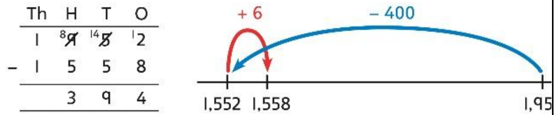
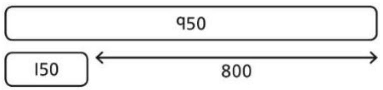
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="3">**Year 5 Subtraction**</td><td>**Representing subtractions and checking strategies**</td><td></td><td>Use bar models to represent subtractions where a part needs to be calculated.

“I can work out the total number of Yes votes using 5,762 − 2,899.”

Bar models can also represent ‘find the difference’ as a subtraction problem.
</td><td>Use inverse operations to check subtractions.

“I calculated 1,225 − 799 = 574. I will check by adding the parts.”

“The parts do not add to make 1,225. I must have made a mistake.”</td></tr>
<tr><td>See Year 3</td><td colspan="3">Use **column subtraction methods** with exchange where required.</td></tr>
<tr><td>**Checking strategies and representing subtractions**</td><td></td><td>Bar models represent subtractions in problem contexts, including ‘find the difference’.
</td><td>Children should explain the mistake made when the columns have not been ordered correctly.

Use approximation to check calculations.

“I calculated 18,000 + 4,000 mentally to check my subtraction.”</td></tr>
</table>

	Objective	Concrete	Pictorial	Abstract
	Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on. *2,002 − 1,995 =?*  Use addition to check subtractions. *“I calculated 7,546 − 2,355 = 5,191. I will check using the inverse.”*
	Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 m − $\boxed{}$ m = $\boxed{}$ m *1 − 0·49 =?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 =?*  Exchange 1 tenth for 10 hundredths.  Now subtract the 5 hundredths.  Now subtract the 2 tenths, then the 2 ones. 	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 − 3·75 =?* O · Tth Hth Thth 3 · 9 2 1 − 3 · 7 5 0 ·

	Objective	Concrete	Pictorial	Abstract
Year 6 Subtraction	See Year 3-5	Use **column subtraction methods** with exchange where required.		
	Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers. 	Compare subtraction methods alongside place value representations.   $\begin{array}{rcccc} & Th & H & T & O \\ & 2 & 6 & 7 & 9 \\ - & & 5 & 3 & 4 \\ \hline & 2 & 1 & 4 & 5 \end{array}$ Use a bar model to represent calculations, including 'find the difference' with two bars as comparison. 	Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy.  Use column subtraction for decimal problems, including in the context of measure. $\begin{array}{rccccc} & H & T & O & \cdot\ Tth & Hth \\ \hline & 3 & 0 & 9 & \cdot\ 6 & 0 \\ - & 2 & 0 & 6 & \cdot\ 4 & 0 \\ \hline & 1 & 0 & 3 & \cdot\ 2 & 0 \end{array}$
	Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 − 150,000* *"That is 950 thousands − 150 thousands*  *So, the difference is 800 thousands."* *950,000 − 150,000 = 800,000*	Subtract efficiently from powers of 10. *10,000 − 500 =?*

Multiplication and Division

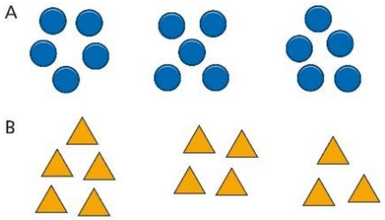
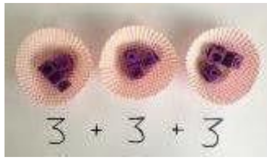
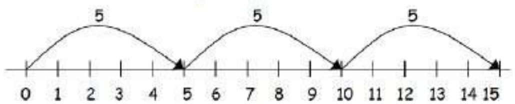
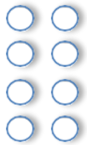
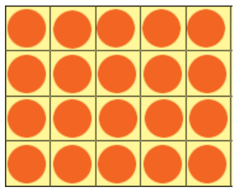
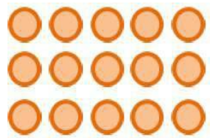
Key Stage 1	Lower Key Stage 2	Upper Key Stage 2
Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.	Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.	Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.

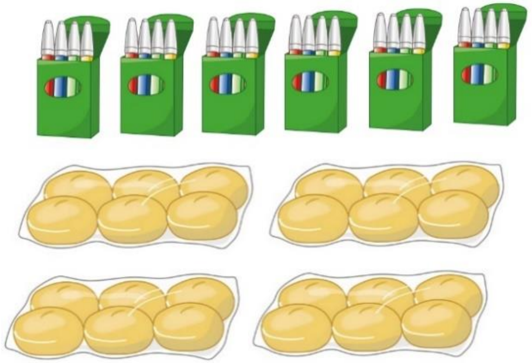
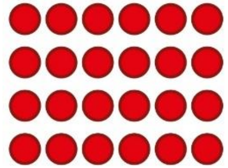
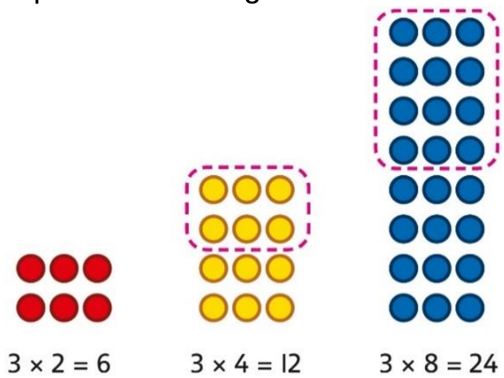
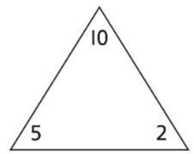
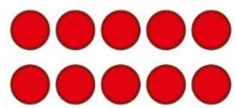
MULTIPLICATION

Vocabulary linked to Multiplication:
Repeated addition, equal groups, groups of, lots of, multiply, times, multiplication, multiplied by, product, array, prime number, square number, cube number

Common misconceptions:

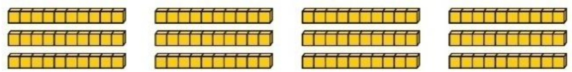
- Understanding on multiplying by 10/100 and what happens to place value of the number
- Rapid recall of multiplication tables is not secure and impacts on the accuracy of calculation
- Interpretation of digits in the T/H columns as single digits e.g. 4x3 instead of 4x30

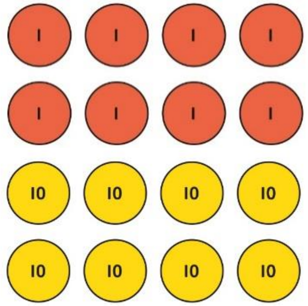
	Objective	Concrete	Pictorial	Abstract
Year 1 Multiplication	**Recognising and making equal groups**	Arrange objects in equal and unequal groups and understand how to recognise whether they are equal.  	Children draw and represent equal and unequal groups.  	Describe equal groups using words *"Three equal groups of 4.* *Four equal groups of 3."*
Year 2 Multiplication	**Repeated addition**	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 + 2 + 2 = 6  5 + 5 + 5 = 15	Write repeated addition sentences to describe objects and pictures.  2 + 2 + 2 = 6
	Arrays- showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8  Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

	Objective	Concrete	Pictorial	Abstract
Year 3 Multiplication	**Using commutativity to support understanding of the times-tables**	Understand how to use times-tables facts flexibly.  There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. *“I can use 6 × 4 = 24 to work out both totals.”*	Understand how times-table facts relate to commutativity.  *6 × 4 = 24* 4 × 6 = 24	Explain how times-table facts relate to commutativity. *“I need to work out 4 groups of 7.* *I know that 7 × 4 = 28 so, I know that* *4 groups of 7 = 28 and 7 groups of 4 = 28.”*
	Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as ‘groups of’, but apply their knowledge of commutativity.  *“I can use the ×3 table to work out how many keys.”* *“I can also use the ×3 table to work out how many batteries.”*	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling. 	Children understand the relationship between related multiplication and division facts in known times-tables.   *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*

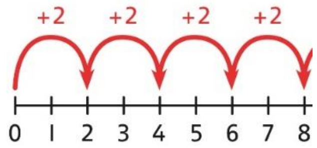
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 3 Multiplication</td>
<td>Using known facts to multiply 10s, for example 3 × 40</td>
<td>Explore the relationship between known times-tables and multiples of 10 using place value equipment.

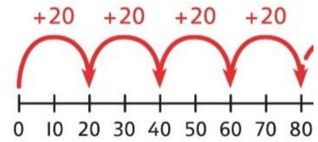
Make 4 groups of 3 ones.

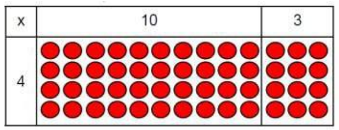
Make 4 groups of 3 tens.


“What is the same? What is different?”</td>
<td>Understand how unitising 10s supports multiplying by multiples of 10.


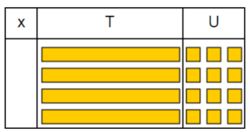
4 groups of 2 ones is 8 ones.
4 groups of 2 tens is 8 tens.

4 × 2 = 8
4 × 20 = 80</td>
<td>Understand how to use known times-tables to multiply multiples of 10.




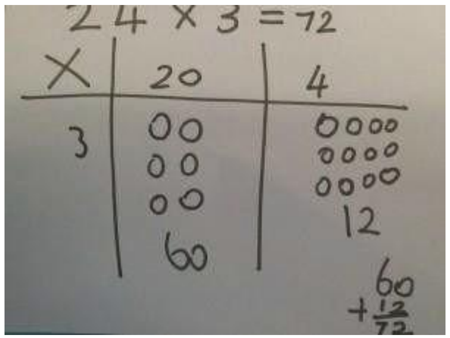
4 × 2 = 8
4 × 20 = 80</td></tr>
<tr><td>Grid method</td>
<td>Show the link with arrays to first introduce the grid method.


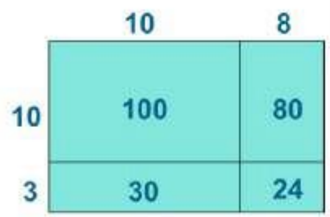
4 rows of 10
4 rows of 3

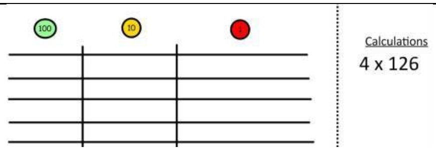
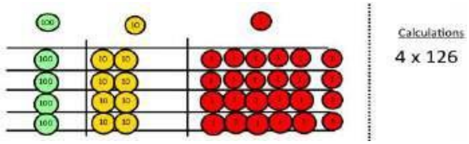
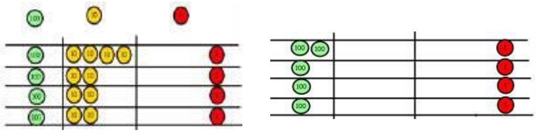
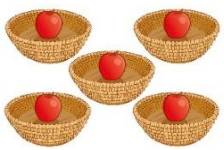
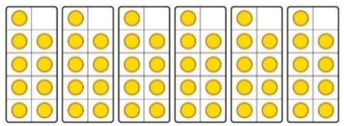
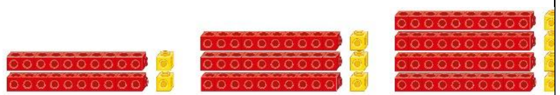
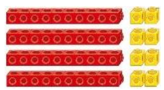
Move on to using dienes to move towards a more compact method.


4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</td>
<td>Children can represent the work they have done with place value counters in a way that they understand.

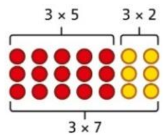
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
<table><tr><th>×</th><th>30</th><th>5</th></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245

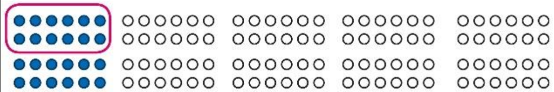
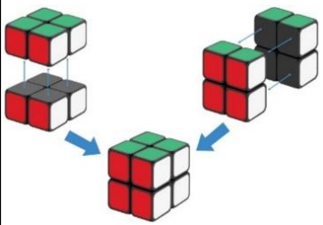
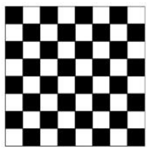
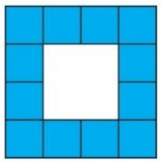
Moving forward, multiply by a 2-digit number showing the different rows within the grid method.

</td></tr>
</table>

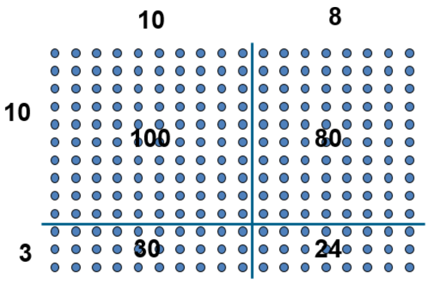
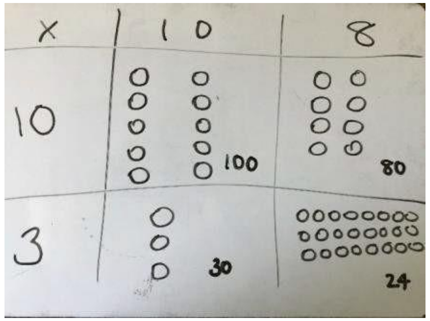
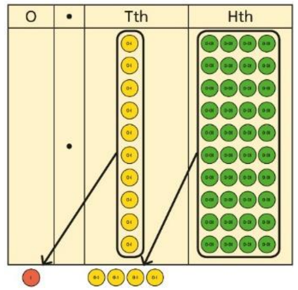
<table>
<tr>
<td></td>
<td></td>
<td>
Fill each row with 126.

Add up each column, starting with the ones making any exchanges needed.

4 x 126 = 504</td>
<td></td>
<td><table><tr><th>X</th><th>1000</th><th>300</th><th>40</th><th>2</th></tr><tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr><tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr></table></td>
</tr>
<tr>
<td>Year 4 Multiplication</td>
<td>Understanding times-tables up to 12 × 12</td>
<td>Understand the special cases of multiplying by 1 and 0.

5 × 1 = 5 5 × 0 = 0</td>
<td>Represent the relationship between the ×9 table and the ×10 table.

Represent the ×11 table and ×12 tables in relation to the ×10 table.

2 × 11 = 20 + 2
3 × 11 = 30 + 3
4 × 11 = 40 + 4

4 × 12 = 40 + 8</td>
<td>Understand how times-tables relate to counting patterns.

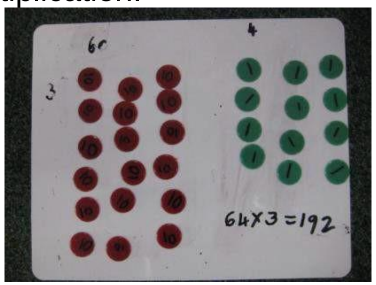
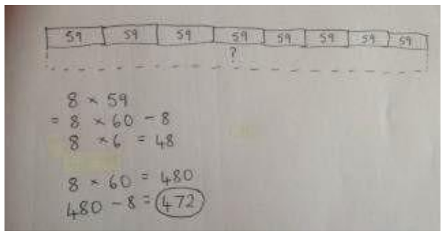
Understand links between the ×3 table, ×6 table and ×9 table
5 × 6 is double 5 × 3

×5 table and ×6 table
“I know that 7 × 5 = 35
so I know that 7 × 6 = 35 + 7.”

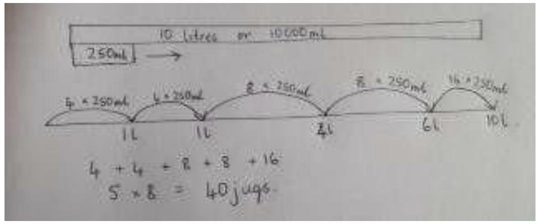
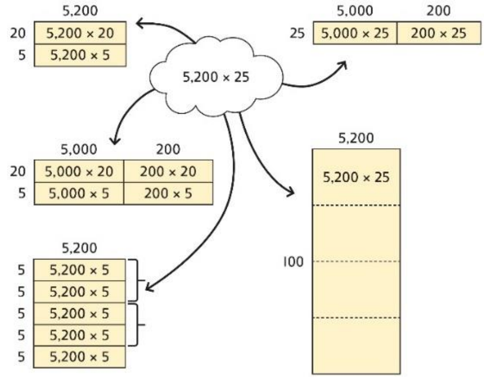
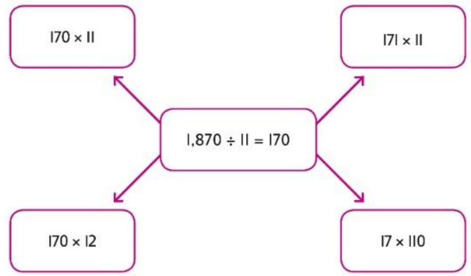
×5 table and ×7 table
3 × 7 = 3 × 5 + 3 × 2

×9 table and ×10 table
6 × 10 = 60
6 × 9 = 60 − 6</td>
</tr>
</table>

	Objective	Concrete	Pictorial	Abstract
Year 4 Multiplication	See Year 3	Use **Grid method**		
Year 4 Multiplication	**Multiplying more than two numbers**	Represent situations by multiplying three numbers together.  Each sheet has 2 × 5 stickers. There are 3 sheets. There are 5 × 2 × 3 stickers in total. $\underbrace{5 \times 2}_{10 \times 3 = 30} \times 3 = 30$	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* *12 × 10 = 120* *10 × 6 × 2 = 120* *60 × 2 = 120*	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5* $12 \times \underbrace{2 \times 5}_{} =$ $12 \times 10 = 120$ So, 24 × 5 = 120
Year 5 Multiplication	**Understanding factors**	Use cubes or counters to explore the meaning of 'square numbers'. *"25 is a square number because it is made from 5 rows of 5."* Use cubes to explore cube numbers.  8 is a cube number.	Use images to explore examples and non-examples of square numbers.  *8 × 8 = 64* *8^2 = 64*  "12 is not a square number, because you cannot multiply a whole number by itself to make 12."	Understand the pattern of square numbers in the multiplication tables. Use a multiplication grid to circle each square number. Can children spot a pattern?

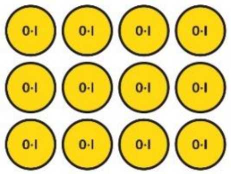
<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Year 5 Multiplication</td><td>Expanded method</td><td>Show the link with arrays to first introduce the expanded method.

</td><td>
</td><td>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.
18
x 13
24 (3 x 8)
30 (3 x 10))
80 (10 x 8)
100 (10 x 10)
234</td></tr>
<tr><td></td><td>Multiplying decimals by 10, 100 and 1,000</td><td>Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.</td><td>Represent multiplication by 10 as exchange on a place value grid.


0·14 × 10 = 1·4</td><td>Understand how this exchange is represented on a place value chart.
<table><tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th><th>•</th><th>Tth</th></tr><tr><td></td><td></td><td></td><td></td><td>2</td><td>•</td><td>5</td></tr><tr><td>2·5 × 10 = 25</td><td></td><td></td><td>2</td><td>5</td><td>•</td><td></td></tr><tr><td>2·5 × 100 = 250</td><td></td><td>2</td><td>5</td><td>0</td><td>•</td><td></td></tr><tr><td>2·5 × 1,000 = 2,500</td><td>2</td><td>5</td><td>0</td><td>0</td><td>•</td><td></td></tr></table></td></tr>
<tr><td>Year 6 Multiplication</td><td>Compact method</td><td>Children can continue to be supported by place value counters at this stage of multiplication.

</td><td>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

</td><td>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

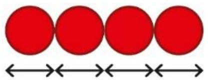
If it helps, children can write out what they are solving next to their answer.
<table><tr><td></td><td></td><td></td><td>7</td><td>4</td></tr><tr><td></td><td>×</td><td></td><td>6</td><td>3</td></tr><tr><td></td><td></td><td></td><td>1</td><td>2</td></tr><tr><td></td><td></td><td>2</td><td>1</td><td>0</td></tr><tr><td></td><td></td><td>2</td><td>4</td><td>0</td></tr><tr><td>+</td><td>4</td><td>2</td><td>0</td><td>0</td></tr><tr><td></td><td>4</td><td>6</td><td>6</td><td>2</td></tr></table></td></tr>
</table>

		It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.		This moves to the more compact method. $\begin{array}{r} 1342 \\ \times\quad 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \end{array}$
Year 6 Multiplication	**Using knowledge of factors and partitions to compare methods for multiplications**	Use equipment to understand square numbers and cube numbers.  *$5 \times 5 = 5^2 = 25$* *$5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$*	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately.  Represent and compare methods using a bar model.	Use a known fact to generate families of related facts.  Use factors to calculate efficiently. *15×16* *$= 3 \times 5 \times 2 \times 8$* *$= 3 \times 8 \times 2 \times 5$* *$= 24 \times 10$* *$= 240$*

<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Year 6 Multiplication</td>
<td>Multiplying decimals</td>
<td>Explore decimal multiplications using place value equipment and in the context of measures.

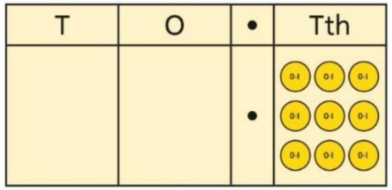


3 groups of 4 tenths is 12 tenths.
4 groups of 3 tenths is 12 tenths.

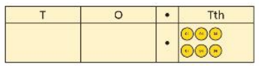


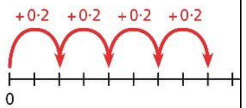
4 × 1 cm = 4 cm
4 × 0·3 cm = 1.2 cm
4 × 1·3 = 4 + 1·2 = 5·2 cm</td>
<td>Represent calculations on a place value grid.

3 × 3 = 9
3 × 0·3 = 0·9



Understand the link between multiplying decimals and repeated addition.




</td>
<td>Use known facts to multiply decimals.

4 × 3 = 12
4 × 0·3 = 1·2
4 × 0·03 = 0·12

20 × 5 = 100
20 × 0·5 = 10
20 × 0·05 = 1

Find families of facts from a known multiplication.

I know that 18 × 4 = 72.
This can help me work out:

1·8 × 4 =?
18 × 0·4 =?
180 × 0·4 =?
18 × 0·04 =?

Use a place value grid to understand the effects of multiplying decimals.

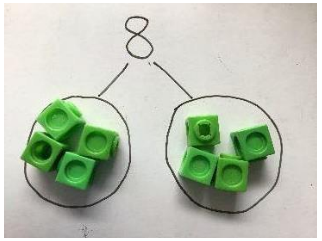
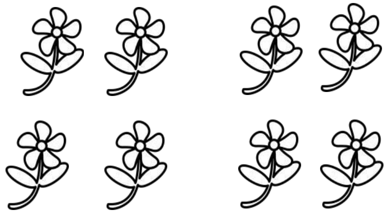
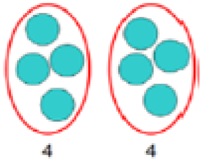
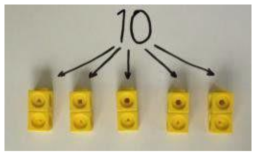
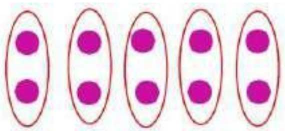
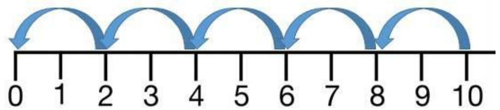
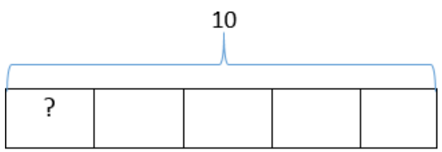
<table><tr><th></th><th>H</th><th>T</th><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>2 × 3</td><td></td><td></td><td>6</td><td>•</td><td></td><td></td></tr><tr><td>0·2 × 3</td><td></td><td></td><td>0</td><td>•</td><td>6</td><td></td></tr><tr><td>0·02 × 3</td><td></td><td></td><td></td><td>•</td><td></td><td></td></tr></table></td>
</tr>
</table>

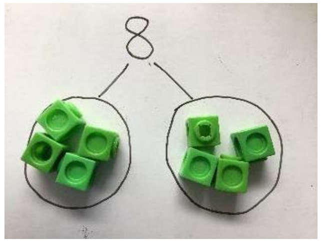
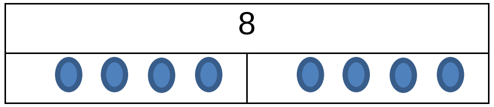
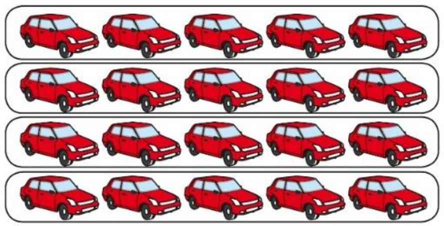
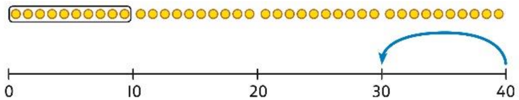
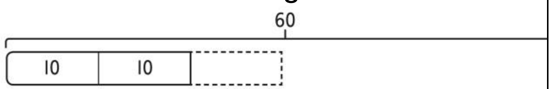
DIVISION

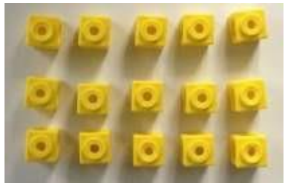
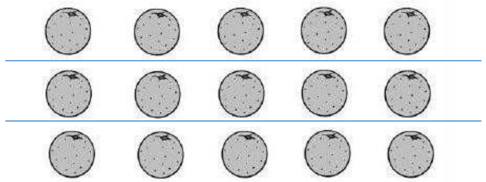
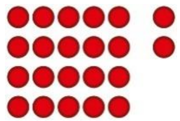
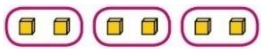
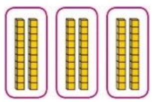
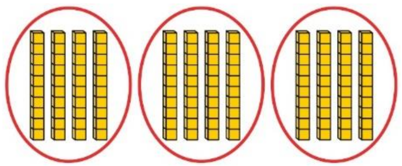
Vocabulary linked to Division:
divisor, divisible, divide, group, grouping, share, chunk, remainder, sharing, shared equally, equal groups, factor, multiple, prime number, square number, cube number, bar model

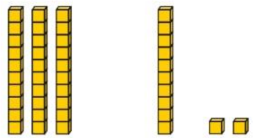
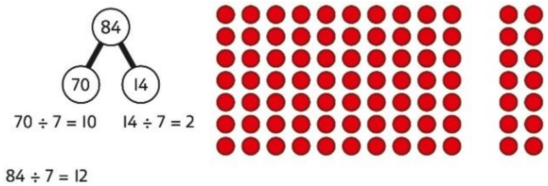
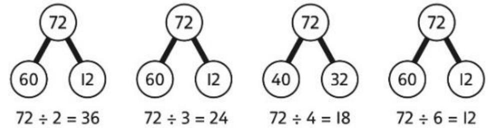
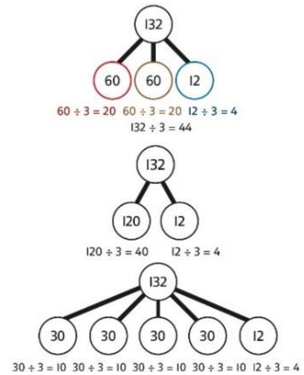
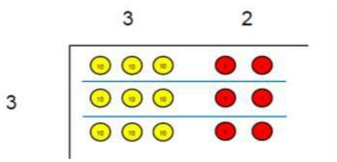
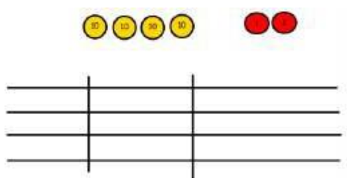
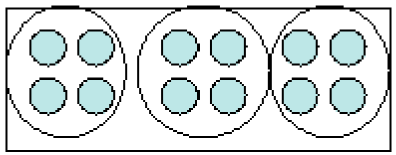
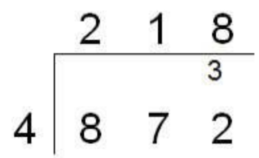
Common misconceptions:

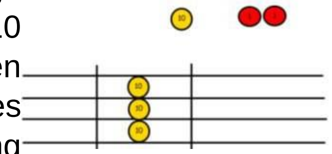
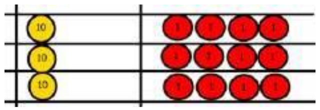
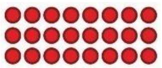
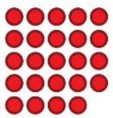
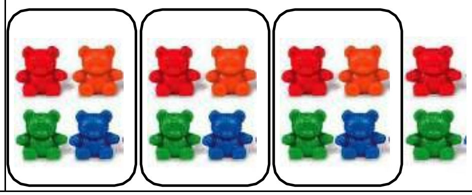
- Lack of understanding of ‘remainders’ and their importance to the context of the problem.
- Insecure understanding of place value to know what each digit is representing.
- Unable to derive facts from known facts and ‘play’ with numbers.
- Approximations are wildly inaccurate so answers cannot be judged in the context of the problem/calculation.
- No method to ‘fall back’ on where use of a formal method won’t work.

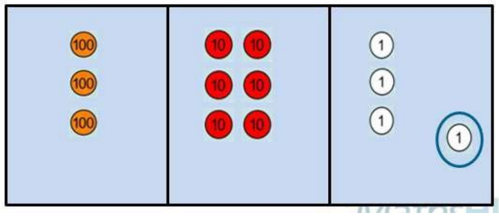
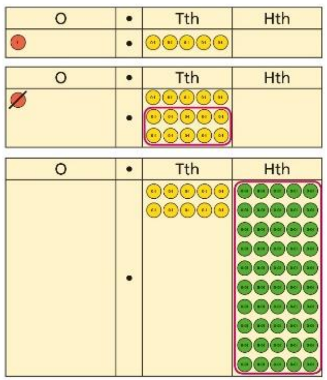
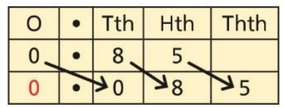
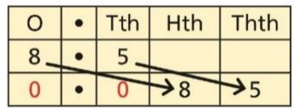
	Objective	Concrete	Pictorial	Abstract
Year 1 Division	**Sharing**	I have 8 cubes; can you share them equally between two people?  	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  	Share 8 buns between two people. 8 ÷ 2 = 4
	Grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  10 ÷ 5 = ? 5 x ? = 10	10 ÷ 5 = 2 Divide 10 into 5 groups. How many are in each group?

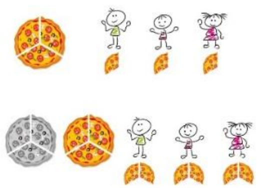
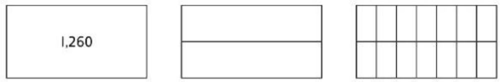
	Objective	Concrete	Pictorial	Abstract
Year 2 Division	**Sharing**	I have 8 cubes; can you share them equally between two people? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  	Share 8 buns between two people. 8 ÷ 2 = 4
	Grouping See Year 1			
	Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *"4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5."*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *"40 divided by 4 is 10."* Use a bar model to support understanding of the link between times-table knowledge and division.  	Relate times-table knowledge directly to division. 1 × 10 = 10 2 × 10 = 20 **3 × 10 = 30** 4 × 10 = 40 5 × 10 = 50 6 × 10 = 60 7 × 10 = 70 8 × 10 = 80 I used the 10 times table to help me. 3 x 10 = 30 *"I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3."* 3 × 10 = 30 so 30 ÷ 10 = 3

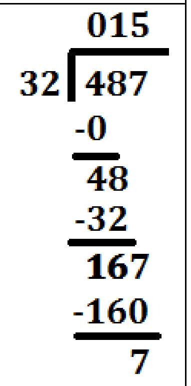
	Objective	Concrete	Pictorial	Abstract
Year 3 Division	**Division with arrays**	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  e.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5
	Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  *"There are 13 sticks in total. There are 3 groups of 4, with 1 remainder."*	Use images to explain remainders.  22 ÷ 5 = 4 remainder 2	Understand that the remainder is what cannot be shared equally from a set. 22 ÷ 5 =? 3 × 5 = 15 4 × 5 = 20 5 × 5 = 25 … this is larger than 22 So, 22 ÷ 5 = 4 remainder 2
	Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. Make 6 ones divided by 3.  Now make 6 tens divided by 3.  *"What is the same? What is different?"*	Divide multiples of 10 by unitising.  *"12 tens shared into 3 equal groups. 4 tens in each group."*	Divide multiples of 10 by a single digit using known times-tables. 180 ÷ 3 =? *180 is 18 tens.* *18 divided by 3 is 6.* *18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*

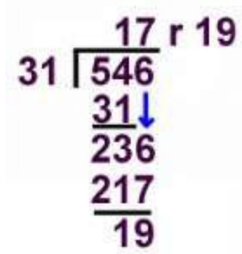
	Objective	Concrete	Pictorial	Abstract
Year 4 Division	**Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning**	Use place value equipment to explore why different partitions are needed. 42 ÷ 3 =? *“I will split it into 30 and 12, so that I can divide by 3 more easily.”* 	Represent how to partition flexibly where needed. 84 ÷ 7 =? *“I will partition into 70 and 14 because I am dividing by 7.”*  	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division. 
	Short division	Use place value counters to divide using the short division method alongside. 96 ÷ 3  42 ÷ 3  Start with the biggest place value. We are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 

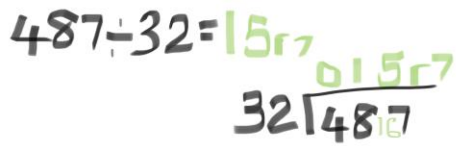
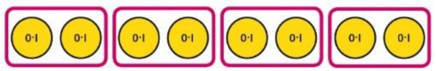
		We exchange this ten for 10 ones and then share the ones equally among the groups.  We look at how many are in each group. 		
Year 5 Division	**Understanding factors and prime numbers**	Use equipment to explore the factors of a given number.  *24 ÷ 3 = 8* *24 ÷ 8 = 3* 8 and 3 are factors of 24 because they divide 24 exactly. 24 ÷ 5 = 4 remainder 4.  5 is not a factor of 24 because there is a remainder.	Understand that prime numbers are numbers with exactly two factors. *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 4 = 4 r 1* 1 and 13 are the only factors of 13. 13 is a prime number.	Understand how to recognise prime and composite numbers. *"I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder."* *"I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33."* *"I know that 1 is not a prime number, as it has only 1 factor."*
	Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder

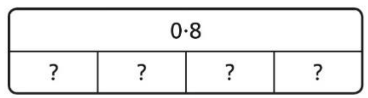
Year 5 Division			Draw dots and group them to divide an amount and clearly show a remainder.  	
	Short division with remainders	364 ÷ 3 = $\begin{array}{r l} & 1\;2\;1 \text{ rem } 1 \\ 3 & \overline{)3\;6\;4} \end{array}$ 		Move onto divisions with a remainder. Once children understand remainders, $\begin{array}{r l} & 8\;\;6\;\;\text{r}\;\;2 \\ 5 & \overline{)4\;\;3\;\;{}^{3}2} \end{array}$ begin to express as a fraction or decimal according to the context. $\begin{array}{r l} & 1\;\;8\;\;6\;1/5 \\ 5 & \overline{)9\;\;{}^{4}3\;\;{}^{3}1} \end{array}$ $\begin{array}{r l} & 1\;\;4\;.\;6 \\ 3\;5 & \overline{)5\;\;1\;\;{}^{16}1\;.\;{}^{21}0} \end{array}$
	Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. *"2 ones are 20 tenths.* *20 tenths divided by 10 is 2 tenths."*	Represent division using exchange on a place value grid.  	Understand the movement of digits on a place value grid.  0·85 ÷ 10 = 0·085  *8·5 ÷ 100 = 0·085*

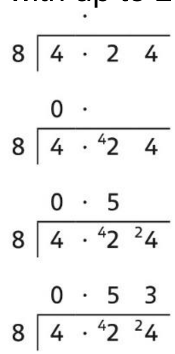
			1·5 is 1 one and 5 tenths. *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.* *1·5 ÷ 10 = 0.15*	
	Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. *“1 whole shared between 3 people. Each person receives one-third.”* 	Use a bar model and other fraction representations to show the link between fractions and division.  $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$
Year 6 Division	**Dividing by a 2-digit number using factors**	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. 1,260 ÷ 14 =?  *1,260 ÷ 2 = 630* *630 ÷ 7 = 90* *1,260 ÷ 14 = 90*	Use factors and repeated division where appropriate. 2,100 ÷ 12 =? 2,100 → $\boxed{\div 2}$ → $\boxed{\div 6}$ → 2,100 → $\boxed{\div 6}$ → $\boxed{\div 2}$ → 2,100 → $\boxed{\div 3}$ → $\boxed{\div 4}$ → 2,100 → $\boxed{\div 4}$ → $\boxed{\div 3}$ → 2,100 → $\boxed{\div 3}$ → $\boxed{\div 2}$ → $\boxed{\div 2}$ →

<table>
<tr><th></th><th>Objective</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 6 Division</td><td>Long division</td><td></td><td></td><td>Children can use long division to divide numbers with up to 4 digits by 2 digit numbers.




Alternatively, if they prefer to use short division, they can just carry the digits across. (see short division- Y4 and Y5)

</td></tr>
<tr><td>Dividing decimals</td><td>Use place value equipment to explore division of decimals.


“8 tenths divided into 4 groups. 2 tenths in each group.”</td><td>Use a bar model to represent divisions.


4 × 2 = 8   8 ÷ 4 = 2
So, 4 × 0·2 = 0·8   0·8 ÷ 4 = 0·2</td><td>Use short division to divide decimals with up to 2 decimal places.

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